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# THE INFLUENCE OF THERMAL DISTORTION ON THE PERFORMANCE OF GRAVITY-STABILIZED SATELLITES

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Moffett Field, Calif.

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#### THE INFLUENCE OF THERMAL DISTORTION ON THE PERFORMANCE

#### OF GRAVITY-STABILIZED SATELLITES

By Gerd Kanning

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#### SUMMARY

An analysis is made of the effect of thermal distortion of booms on the attitude of a gravity-stabilized satellite. The attitude computed when only the effects of thermal distortion on the solar pressure torques were considered is compared with the attitude computed when the interdependent effects of changing geometry and mass distribution were considered as well.

The calculations were made for several satellites having either symmetrical or asymmetrical boom arrangements. Calculations for all arrangements were made for low orbits inclined 45° to the sunline. In addition, calculations were made for one symmetric configuration for synchronous altitude. The results indicated that including the effects of changing geometry and mass distribution is not important for symmetrical arrangements, but is important for asymmetrical boom arrangements. Furthermore, for asymmetrical boom arrangements, it was found that the thermal distortion can provide some compensation for the unbalanced solar pressure torques attributable to asymmetry. Thus, the best pointing accuracy might be achieved with booms designed to have a specific thermal distortion.

#### INTRODUCTION

For gravity stabilization of a satellite large differences in the principal moments of inertia are required to achieve satisfactory restoring torque levels. These differences are usually obtained by erecting one or more booms from the main satellite body, or "stabilized package." The booms are also a source of disturbance arising from solar radiation pressure forces. Careful design of the satellite is required to minimize attitude errors resulting from this disturbance.

The obvious way of minimizing the attitude errors is to minimize the unbalanced solar torques. This can be accomplished, in principle, by (a) erecting the booms symmetrically and (b) constructing the booms so that they do not bend when heated by solar radiation. Condition (a) ensures that the centers of satellite and projected area nominally coincide while condition (b) ensures that condition (a) holds for all orientations of the satellite relative to the sun.

Condition (a) imposes a considerable restriction on the permissible satellite geometry and, therefore, greatly infuences the overall design. The configurations that satisfy condition (a) tend to have a large number of booms since they must be arranged in symmetrically disposed pairs, implying added complication, lower reliability, and possibly greater stabilization system mass than asymmetrical arrangements. If the attitude performance requirements of the satellite are not too stringent, the reduced complication of an asymmetrical boom arrangement may be more important than the decrease in attitude errors gained by minimizing solar torques. This is the situation for low altitude satellites for which the gravity gradient is relatively large and the attitude penalties for unbalanced solar torques more tolerable. It may, therefore, be advantageous from the aspect of overall design not to attempt to meet condition (a).

It is important to recognize here that failure to meet condition (a) may invalidate condition (b) as a means of minimizing attitude errors. As an illustration, consider the static equilibrium state of the simple, asymmetric, single boom satellite shown in figure 1. Although the thermal distortion changes the magnitude of the unbalanced solar torques acting on this satellite, the effect is small compared with that resulting from the basic geomet-

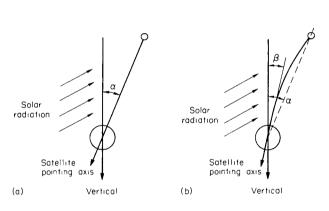


Figure 1.- Simple asymmetric two-axis gravity-stabilized satellite.

rical asymmetry. Therefore, if the mass of the boom is small compared with the masses of both the tip and the stabilized package, the angle  $\alpha$ between the vertical and the line connecting the tip mass to the stabilized package (fig. 1(a)) is only slightly influenced by thermal distortion. But thermal distortion (provided it is not too great) rotates the stabilized package so that the earth-pointing axis, fixed in the stabilized package, is closer to the vertical (fig. 1(b), angle  $\beta$ ), and pointing accuracy is improved. This example demonstrates the purely geometrical benefits of thermal distortion.

These geometric effects on asymetrical satellites are not the only benefits of thermal distortion. It also changes the mass distribution of some satellites so as to reduce pointing errors. Consider the static equilibrium state of the simple, asymmetric, two-boom satellite shown in figure 2, whose pointing axis is normally parallel to the principal axis of inertia. With the solar radiation in the direction indicated, boom A experiences practically no solar pressure, whereas boom B is almost orthogonal to the radiation and experiences the maximum solar pressure. Relatively large unbalanced solar torques therefore act on the satellite. In equilbrium, the principal axis of inertia makes an angle with the local vertical, and for a satellite without thermal distortion, the pointing axis makes the same angle. On the other hand, thermal distortion rotates the principal axis of inertia so that it is no longer parallel to the pointing axis. Since thermal distortion has only

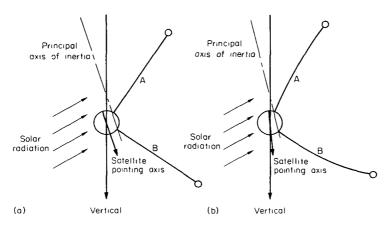


Figure 2.- Simple asymmetric three-axis stabilized satellite.

a minor influence on the solar pressure torques, it is clear from figure 2 that the pointing axes must be rotated toward the local vertical. Pointing accuracy is again improved.

The major objection to these arguments is that, except for certain special orbits and satellite configurations, static equilibrium never occurs. In general, since the thermal distortion varies with position of the satellite in the orbit, the mass distribution varies continuously and rigid-body dynamics do not apply. An adequate dynamical model must represent not only the rates of change of inertia but also the rates of change of the angular momentum of the distorting portions of the satellite relative to the rigid stabilized package. The simple arguments, based on static equilibrium, nevertheless, give the intuitive feeling that attitude errors can be reduced by permitting a controlled amount of thermal distortion. This possibility merits investigation in view of recent advances in boom technology, which allow the designer much greater control over thermal bending.

Flexural motions of booms distinct from the thermal distortion described above can also be caused by solar heating. Two types of flexural motion have been observed, neither of which is considered in this study. One is the result of instability caused by steady heating. This instability can produce disturbances far exceeding those resulting from the type of boom motion considered in this study (see ref. 2). However, recent improvements in boom design should eliminate this type of flexural motion (see ref. 3). The second type of flexural motion is caused by sudden temperature changes as the satellite moves into and out of the earth's shadow (thermal twang). For the satellites under consideration, the period of these oscillations will be much shorter than the period of the librational motion. Therefore, little coupling will exist and the librational and flexural modes can be analyzed separately.

In this paper, the influence of thermal distortion on one symmetric and two asymmetric satellite configurations is examined (fig. 3). All three configurations are stabilized about all axes and are of the coupled, single damper type (ref. 1). In both asymmetrical configurations the fixed booms form a symmetrical V fixed to the stabilized package at the apex. In one configuration the V is bisected by the local vertical and points earthward. In

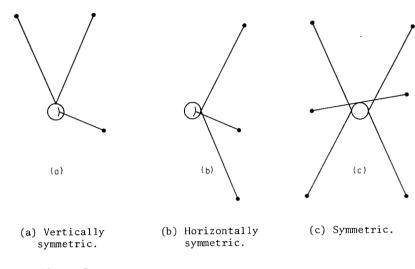


Figure 3.- Inertially coupled, three-axis gravity-oriented satellites.

the other it is bisected by the local horizontal plane and points approximately in the direction of the velocity vector. These satellites are termed the vertically and horizontally symmetric satellites, respectively (fig. 3). The study is directed at determining the variation of attitude errors due to solar pressure torque alone, over a range of values of the radius of curvature of the thermally distorted booms.

It has been customary in past studies of thermally distorted symmetrical satellites to assume that the dynamic effects of variable mass distribution are small (ref. 4). The argument for neglecting these terms from the equations of motion is based on the nominal symmetry of the configuration and the symmetry of the distorted shape. The validity of this assumption is also examined in this paper.

#### NOMENCLATURE

В	damping constant of angular rate damper; made dimensionless by dividing by $\mathbf{I}_{d}\mathbf{W}_{0}$
ъ́ј	unit vectors along the axes of principal moments of inertia of the main satellite body
С	spring constant of mechanical spring connected to the damper boom; made dimensionless by dividing by $~{\rm I_dW_0}^2$
<sup>c</sup> j	unit vectors defining orthogonal reference frame in main satellite body: $\bar{c}_2$ lies along the hinge axis of the damper, $\bar{c}_{\frac{1}{V}}$ lies along the damper boom in equilibrium, and $\bar{c}_3$ lies along $\bar{V}_3$
$\left(\frac{d}{dt}\right)_{b}$	time derivative operator with respect to the body frame
$\left(\frac{d}{dt}\right)_{i}$	time derivative operator with respect to the inertial frame
$dm_d$	element of mass of damper boom
${\rm dm}_{\rm V}$	element of mass of main body
$F_{\boldsymbol{m}}$	force on particle of mass dm

$\overline{I}_{b}$	inertia matrix of the main body
$^{\mathrm{I}}$ d	damper boom moment of inertia about the hinge axis
ōj	unit vectors defining right-hand orthogonal reference frame fixed at satellite center of mass: $\bar{o}_3$ is directed toward the center of the earth; $\bar{o}_1$ is in the direction of motion
R	radius of curvature of a boom to its axis when solar radiation is normal
$\bar{r}_d$	center of mass location of damper
$\bar{\mathtt{r}}_{\mathtt{m}}$	vector from center of mass of the main body to particle of mass dm
roll, pitch, yaw	attitude angles relating $\overline{V}_j$ frame to $\bar{o}_j$ frame; roll is a rotation about the velocity vector; pitch is a negative rotation about the angular velocity vector, and yaw is a rotation about the earth-pointing axis
$\overline{\mathrm{T}}$	total torque on main body
T <sub>d/b</sub>	torque on the damper boom due to the main body
T <sub>e/b</sub>	external torques on the body
<sup>T</sup> e/d	external torques on the damper
$\overline{\overline{\mathbb{U}}}$	unit dyadic operator
$\overline{v}_{j}$	unit vectors defining orthogonal reference frame fixed in main satellite body; in equilibrium, $\overline{V}_j = \bar{o}_j$
$V_{\rm m}$	velocity of particle $dm$ ; given by $\frac{dr_m}{dt}$
Υ	damper offset angle; angle between $V_2$ and $\bar{c}_2$ axes
$\omega_{\mathbf{o}}$	orbital angular rate
- ω̄b/i	angular rate of body with respect to the inertial frame
$^{\bar{\omega}}$ d/i	angular velocity of damper body with respect to inertial space

#### ANALYSIS

The effects of thermal distortion were analyzed by comparing the calculated steady-state response when thermal distortion was assumed to influence the solar pressure torques alone, and when it was also assumed to cause variations in geometry and the mass distribution. The steady-state motion was calculated in each case by a digital integration of the equations of motion

over a sufficient time to permit transient motion to become negligible. The scalar equations that were integrated were expanded from the vector equations of motion given in the appendix A.

In calculating the variation of the mass distribution caused by thermal distortion, the following assumptions were made concerning the bending of the booms:

- 1. Curvature was assumed to be constant throughout the length of any boom.
- 2. The curvature for a given solar incidence was identical for all booms and was proportional to the absorbed energy per unit length.
  - 3. The plane defined by a curved boom contains the sunline.
- 4. A boom reaches its thermal equilibrium position instantaneously following a change in illumination.

The last assumption eliminates any consideration of flexural modes of motion.

The analysis was limited to a study of several specific satellite configurations. The configurations were related in that each relied solely upon gravity for both stabilization and damping. The principal difference between the various configurations was the number or orientation of the booms. Details of the geometry, mass distribution, and orbits are given in table I, and sketches of each are shown in figure 3.

It was assumed in calculating the attitude motion that disturbances from solar radiation were the only source of attitude error. The calculated attitude errors therefore are not representative of the total error that might exist for an actual satellite. Most of the results are for satellites assumed to be in a 1200 km orbit inclined 45° to the sunline. For this particular orbit, the satellite passes through the earth's shadow. The effects of thermal distortion on a symmetrical satellite in a synchronous equatorial orbit at the time of a solstice were also studied.

#### RESULTS AND DISCUSSION

#### Asymmetrical Satellites

The boom arrangements for the asymmetrical satellites were chosen to satisfy a given set of moment of inertia ratios with no consideration given to minimizing solar disturbances or the response to them. Comparisons of the pointing accuracies of the two asymmetrical satellites therefore have little meaning since either might be improved considerably, and either might be considerably different if the inclination of the orbit to the sun were changed.

The influence of the variation in mass distribution caused by thermal distortion on the attitude motion was quite different for the two configurations studied as is illustrated in figure 4. For these particular results, the booms were representative of the nonperforated, silver-plated, berylliumcopper booms which have a curvature when normal to the sunline of about 457 m. For the vertically symmetric satellite, the maximum error in earth pointing was underestimated by a factor of about 5, and the error in yaw by a factor of about 2 when thermal distortion was assumed to influence only the solar pressure torque. In contrast, including all the effects of thermal distortion was not as important in calculating the attitude motion of the horizontally symmetric satellite. For this satellite, including all the effects changed the phase but not the magnitude of the small pitch and roll errors (see fig. 4(b)). The calculated yaw attitude, however, was sensitive to inclusion of all of the effects of distortion. Inclusion of all of the effects decreased the amplitude of the calculated yaw motion and indicated a change in the yaw bias of about -2°.

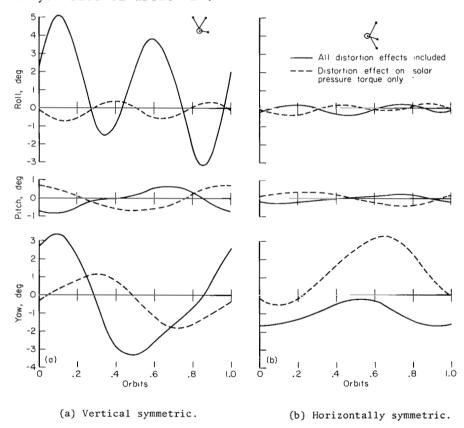


Figure 4.- The effect of thermal distortion on the performance of an asymmetrical satellite; R = 457 m.

In the Introduction it was argued that a specified amount of thermal distortion might reduce the attitude errors. This possibility was investigated by varying the boom curvature, R, due to thermal distortion, and comparing the root-mean-square (RMS) roll, pitch, and yaw errors. These results

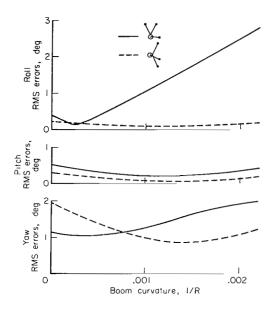


Figure 5.- RMS pointing error as a function of boom curvature.

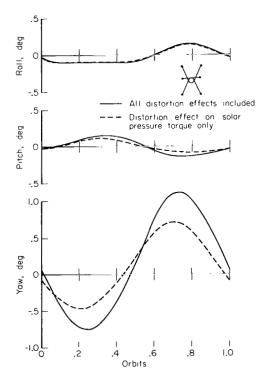


Figure 6.- Steady-state performance for a
low altitude symmetric satellite;
R = 457 m.

are summarized in figure 5 where it is shown that minima do exist for a boom curvature other than zero. For the two examples given, the largest effect of varying the curvature is to reduce the RMS roll angle of the vertically symmetric satellite from 3° for a typical nonperforated silver-plated boom (1/R = 0.0022 m<sup>-1</sup>) to nearly zero for a boom with nearly zero curvature. For this particular example, the engineering choice would probably be a boom designed to have zero curvature.

For the horizontally symmetric satellite, the minimum RMS errors occur at a rod curvature of about 0.0014 m<sup>-1</sup>. The principal effect of changing curvature for this satelite is a change in the yaw error. Detailed examination of the time histories from which the results of figure 6 were derived shows that the minimum RMS yaw error occurs when the steady component of the yaw error is zero. It is anticipated that a change in the orbit relation to the sunline might alter this steady yaw bias and the boom curvature for which it is zero.

It must be emphasized that all of the results presented are considered to be isolated examples. The only conclusion that can be drawn is that all of the effects of thermal distortion must be included when the performance of gravity stabilized satellites with asymmetrical boom arrangements is estimated. In addition, the possibility of minimizing the errors through specification of boom curvature should be investigated.

#### Symmetrical Satellite

It has been assumed in past performance studies of symmetrical gravity stabilized satellites that the primary effect of thermal distortion was its influence on solar pressure torques (see ref. 4). The argument supporting this assumption is that distortion will always result primarily in translation of the principal axes relative to the stabilized package and relative to the orbital reference axes.

Symmetrical gravity stabilized satellites are considered primarily for high altitude orbits (such as synchronous orbits) where asymmetry cannot be

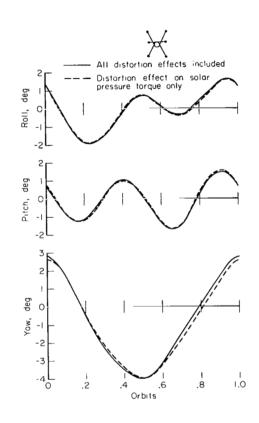


Figure 7.- The effect of thermal distortion on synchronous altitude.

tolerated because of large unbalanced solar-pressure torques. For this reason, the effects of thermal distortion were studied for a synchronous satellite as well as for a satellite in a 1200 km orbit. At the lower altitude, the attitude errors are small (fig. 6). The errors in earth pointing and in yaw are underestimated by only about 0.1° and 0.5°, respectively, when distortion is assumed to influence solar pressure torque only. Even though the difference in the two estimates is small, it represents a large fraction of the total error. For a synchronous satellite, the situation is quite different. At this altitude the gravity gradient is greatly reduced and the ratio of solar disturbance torque to restoring torque is increased because of restrictions placed on the mass that can be allotted to the stabilization system. A corresponding increase occurs in the importance of the solar pressure torque caused by thermal distortion compared to the other effects. This is illustrated in figure 7 where the calculated motion is shown for the satellite with the characteristics listed in table I. In this instance, the error attributable to solar pressure torque alone is nearly identhe performance of a symmetric satellite at ical to that calculated when all of the distortion effects are included.

#### CONCLUSIONS

Thermal distortion of the booms of a gravity stabilized satellite influences the attitude of the satellite not only by causing solar pressure torques, but by changing the satellite geometry and mass distribution. changes, while relatively unimportant for a symmetrical satellite, can be as important for an asymmetrical satellite as the solar pressure torques. thermore, minimum pointing errors for asymmetrical satellites may occur with booms that have a nonzero thermally induced curvature.

Ames Research Center National Aeronautics and Space Administration Moffett Field, California 94035, July 22, 1969

#### APPENDIX A

#### DEVELOPMENT OF THE VECTOR EQUATIONS OF MOTION

#### FOR GRAVITY-STABILIZED SATELLITE

The equations of motion for the gravity-stabilized satellite configurations studied are developed here in vector form. Consider the sketch in figure 8. The force on particle dm is given by

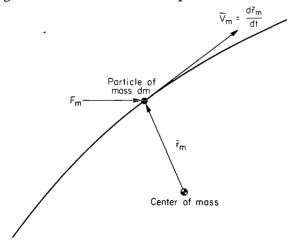


Figure 8.- A particle of mass dm moving in some arbitrary path.

$$\overline{F}_{m} = dm \frac{d\overline{V}_{m}}{dt}$$
 (1)

Then the torque on the satellite will be:

$$\overline{T} = \sum_{m} \overline{r}_{m} \times \overline{F}_{m}$$
 (2)

If equation (1) is substituted into (2)

$$\overline{T} = \sum_{m} \overline{r}_{m} \times dm \frac{d\overline{V}_{m}}{dt} = \sum_{m} dm \left( \overline{r}_{m} \times \frac{d\overline{V}_{m}}{dt} \right)$$
(3)

The time derivative of  $(\bar{\mathbf{r}}_m \times \overline{\mathbf{V}}_m)$  is:

$$\frac{d}{dt} \left( \mathbf{r}_{m} \times \overline{\mathbf{V}}_{m} \right) = \frac{d\overline{\mathbf{r}}_{m}}{dt} \times \overline{\mathbf{V}}_{m} + \overline{\mathbf{r}}_{m} \times \frac{d\overline{\mathbf{V}}_{m}}{dt}$$

or

$$\left(\overline{r}_{m} \times \frac{d\overline{V}_{m}}{dt}\right) = \frac{d}{dt} \left(r_{m} \times \overline{V}_{m}\right) - \frac{d\overline{r}_{m}}{dt} \times \overline{V}_{m}$$
(4)

Substitute equation (4) into equation (3)

$$\overline{T} = \sum_{m} dm \left[ \frac{d}{dt_{i}} \left( \overline{r}_{m} \times \overline{V}_{m} \right) - \frac{d\overline{r}_{m}}{dt} \times \overline{V}_{m} \right]$$
(5)

but

$$\frac{d\bar{r}_{m}}{dt} \times \bar{V}_{m} = 0$$

then equation (5) becomes

$$\overline{T} = \sum_{m} dm \frac{d}{dt_{i}} \left( \overline{r}_{m} \times \overline{V}_{m} \right)$$
(6)

The velocity  $\overline{V}_{m}$  of the particle with respect to the inertial frame can be expressed by the kinematic relation:

$$\overline{V}_{m} = \begin{pmatrix} d\overline{r}_{m} \\ dt \end{pmatrix}_{i} = \begin{pmatrix} d\overline{r}_{m} \\ dt \end{pmatrix}_{b} + \omega_{b/i} \times \overline{r}_{m}$$
(7)

If equation (7) is substituted into (6)

$$\overline{T} = \sum_{m} dm \frac{d}{dt_{i}} \left\{ \left[ \overline{r}_{m} \times \left( \frac{d\overline{r}_{m}}{dt} \right)_{b} \right] + \overline{r}_{m} \times \left( \overline{\omega}_{b/i} \times \overline{r}_{m} \right) \right\}$$

$$\overline{T} = \sum_{m} dm \frac{d}{dt_{i}} \left[ \overline{r}_{m} \times \left( \frac{d\overline{r}_{m}}{dt} \right)_{b} \right] + \sum_{m} dm \frac{d}{dt_{i}} \left[ \overline{r}_{m} \times \left( \overline{\omega}_{b/i} \times \overline{r}_{m} \right) \right]$$
(8)

If the first term of equation (8) is expanded

$$\sum_{\mathbf{m}} d\mathbf{m} \frac{d}{d\mathbf{t}_{i}} \left[ \bar{\mathbf{r}}_{\mathbf{m}} \times \left( \frac{d\bar{\mathbf{r}}_{\mathbf{m}}}{d\mathbf{t}} \right)_{\mathbf{b}} \right] = \sum_{\mathbf{m}} d\mathbf{m} \left\{ \bar{\mathbf{r}}_{\mathbf{m}} \times \left( \frac{d^{2}\mathbf{r}_{\mathbf{m}}}{d\mathbf{t}^{2}} \right)_{\mathbf{b}} + \bar{\omega}_{\mathbf{b}/i} \times \left[ \bar{\mathbf{r}}_{\mathbf{m}} \times \left( \frac{d\bar{\mathbf{r}}_{\mathbf{m}}}{d\mathbf{t}} \right)_{\mathbf{b}} \right] \right\}$$
(9)

Expanding the second term of equation (8) yields

$$\sum_{m} dm \frac{d}{dt_{i}} \left[ \bar{r}_{m} \times \left( \bar{\omega}_{b/i} \times \bar{r}_{m} \right) \right] = \sum_{m} dm \frac{d}{dt_{i}} \left( \bar{r}_{m} \cdot \bar{r}_{m} \bar{\bar{U}} - \bar{r}_{m} \bar{r}_{m} \right) \cdot \bar{\omega}_{b/i} \quad (10)$$

where  $\bar{\bar{\mathbb{D}}}$  is the unit dyadic operator. The torque equation can now be written as

$$\overline{T} = \sum_{m} dm \frac{d}{dt_{i}} \left( \overline{r}_{m} \cdot \overline{r}_{m} \overline{\overline{U}} - \overline{r}_{m} \overline{r}_{m} \right) \cdot \overline{\omega}_{b/i} 
+ \sum_{m} dm \left\{ \overline{r}_{m} \times \left( \frac{d^{2} \overline{r}_{m}}{dt^{2}} \right)_{b} + \overline{\omega}_{b/i} \times \left[ \overline{r}_{m} \times \left( \frac{d \overline{r}_{m}}{dt} \right) \right] \right\}$$
(11)

The first part of equation (11) is the inertia dyadic for a system of mass particles that can be expressed as  $d(I_b\overline{\omega}_b/i)/dt$ . The second part of equation (11) represents the rate of change of the angular momentum of the distorting portions of the satellite relative to the rigid stabilized package.

The torque  $\overline{T}$  can be separated into the external torques on the body  $\overline{T}_{e/b}$  and the reaction torque  $T_{d/b}$  due to the interaction of the main body

and damper boom. For the main body equation (11) can then be written as

$$\overline{T}_{e/b} - \overline{T}_{d/b} = \frac{d}{dt_{i}} \left( \overline{I}_{b} \cdot \overline{\omega}_{b/i} \right) + \sum_{m_{V}} dm_{V} \left[ \overline{r}_{m} \times \left( \frac{d^{2} \overline{r}_{m}}{dt^{2}} \right) + \overline{\omega}_{b/i} \times \overline{r}_{m} \times \left( \frac{d \overline{r}_{m}}{dt} \right)_{b} \right]$$
(12)

Likewise a similar expression can be obtained for the damper boom.

$$\overline{T}_{e/d} + \overline{T}_{d/b} = \frac{d}{dt_{i}} \left( \overline{I}_{d} \cdot \overline{\omega}_{d/i} \right) + \sum_{m_{d}} dm_{d} \left[ \overline{r}_{d} \times \left( \frac{d^{2} \overline{r}_{d}}{dt^{2}} \right) + \overline{\omega}_{d/i} \times \overline{r}_{d} \times \left( \frac{d\overline{r}_{d}}{dt} \right)_{d} \right] \quad (13)$$

The vector equations (12) and (13) when expanded into scalar form were integrated by a digital computer until the steady state was reached.

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